Completing the square to solve quadratic equations

Steps to complete the square:

Before beginning ensure that the equation is in the form $ax^2+bx+c=o$

1. Take out the coefficient of x^2 (if applicable) and divide all terms by that coefficient
2. Take the constant over to the right hand side
3. Work out: (half coefficient of x)^2
4. Add the value in step 3 to both sides of the equation
5. Factorise the left hand side trinomial into a bracket. The numerical value will be: half coefficient of x.
6. Square-root both sides, remembering to add in the $\pm$
7. Solve for x like a normal equation.

Worked example 1:

Solve for x in the following equation by completing the square:

$\boldsymbol{x^2 + 12x + 30 = -2}$

Change to form $ax^2+bx+c=o$

$x^2 + 12x + 32 = 0$
$x^2 + 12x = -32$
$x^2 + 12x + 36 = -32 + 36$
$(x + 6)^2 = 4$
$x + 6 = \pm 2$
$x = -6 \pm 2$
$x = -4 \text{ or } x = -8$

Step 1: Coefficient of x^2 is 1 so proceed
Step 2: Constant to right side
Steps 3 & 4: (half coefficient of x)^2
Step 5: factorise into one bracket
Step 6: Square-root both sides ($\pm$!)
Step 7: Solve

$(12 \times \frac{1}{2})^2 = (6)^2 = 36$

Worked example 2:

$\boldsymbol{2x^2 + 7x + 6 = 0}$

$x^2 + \frac{7}{2}x + 3 = 0$
$x^2 + \frac{7}{2}x = -3$
$x^2 + \frac{7}{2}x + \frac{49}{16} = -3 + \frac{49}{16}$
$\left(x + \frac{7}{4}\right)^2 = \frac{1}{16}$
$x + \frac{7}{4} = \pm\frac{1}{4}$
$x = -\frac{7}{4} \pm \frac{1}{4}$
$x = -1.5 \text{ or } x = -2$

Step 1: Take out coefficient of x^2

Step 2: Constant to right side

$(\frac{7}{2} \times \frac{1}{2})^2 = (\frac{7}{4})^2 = \frac{49}{16}$

Steps 3 & 4: (half coefficient of x)^2 - add

Step 5: factorise into one bracket

$(\frac{7}{2} \times \frac{1}{2}) = (\frac{7}{4})$

Step 6: Square-root both sides ($\pm$!)

Step 7: Solve

Exercise:

Solve for *p* in the following equations by completing the square:

a) $p^2 + 8p - 6 = 0$

b) $6p = -3p^2 - 2$

c) $p^2 - 12 = -4p$

Answers:

a) $p^2 + 8p - 6 = 0$
$p^2 + 8p = 6$
$p^2 + 8p + 16 = 6 + 16$
$(p + 4)^2 = 22$
$p + 4 = \pm\sqrt{22}$
$p = -4 \pm \sqrt{22}$
$\therefore p = -4 + \sqrt{22} \quad or \quad p = -4 - \sqrt{22}$

b) $6p = -3p^2 - 2$
$3p^2 + 6p + 2 = 0$
$p^2 + 2p + \frac{2}{3} = 0$
$p^2 + 2p = -\frac{2}{3}$
$p^2 + 2p + 1 = -\frac{2}{3} + 1$
$(p + 1)^2 = \frac{1}{3}$
$p + 1 = \pm\sqrt{\frac{1}{3}}$
$p = -1 \pm \sqrt{\frac{1}{3}}$
$p = \frac{-3 + \sqrt{3}}{3}$ or $p = \frac{-3 - \sqrt{3}}{3}$

c) $p^2 - 12 = -4p$
$p^2 + 4p - 12 = 0$
$p^2 + 4p = 12$
$p^2 + 4p + 4 = 12 + 4$
$(p + 2)^2 = 16$
$p + 2 = \pm 4$
$p = -2 \pm 4$
$\therefore p = 2$ or $p = -6$